Version 1.0

# General Certificate of Education
# June 2010

## Mathematics MM04

## Mechanics 4

# Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme and abbreviations used in marking**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

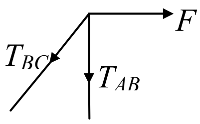
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

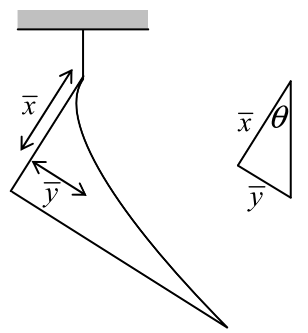
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MM04**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | Moments at $A$: | | | |
| | $50(2\cos 30°) = \text{F}(4\cos 30°)$ | M1 | | One side correct. Use of ratios ok |
| | $\therefore \text{F} = 25\text{N}$ | A1 | 2 | |
| **(b)** | Magnitude 25 N, to the right $(\rightarrow)$ | B1,B1 | 2 | B1 each part |
| **(c)(i)** |   | | | |
| | Resolving horizontally at $B$: | | | |
| | $T_{BC}\ \sin 30° = \text{F}$ | M1 | | Attempt at an equation to find $T_{BC}$ |
| | $T_{BC} = 50\,\text{N}$ | A1F | 2 | ft part (a) |
| **(ii)** | Resolving vertically at $B$: | | | |
| | $T_{AB} + T_{BC}\ \cos 30° = 0$ | M1 | | Attempt at an equation to find $T_{AB}$ |
| | $\lvert T_{AB} \rvert = 25\sqrt{3}$ or $43.3\,\text{N}$ | A1F | 2 | ft part (a); must be positive for A1 |
| | **Total** | | **8** | |
| **2(a)** | Momentum $= I\omega$ | | | |
| | $= 0.6 \times 3$ | M1 | | Evidence of $I\omega$ |
| | $= 1.8\ \text{kg m}^2\,\text{s}^{-1}$ | A1 | 2 | Units required |
| **(b)** | $0.45\omega_1 = 1.8$ | M1 | | Forming equation – conservation of angular momentum |
| | $\omega_1 = 4\,\text{rad s}^{-1}$ | A1F | 2 | ft their part (a); units required |
| | **Total** | | **4** | |

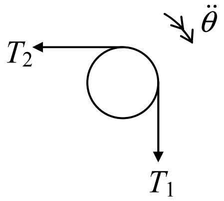
**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 3(a) | Area $= \int_0^2 kx^3\,dx = \left[\frac{kx^4}{4}\right]_0^2$ | M1 | | Attempt to integrate |
| | $= 4k$ | A1 | 2 | |
| (b) | $\int xy\,dx = \int_0^2 kx^4\,dx = \left[\frac{kx^5}{5}\right]_0^2$ | M1 | | Attempt to use $\int xy\,dx$ |
| | $= \frac{32k}{5}$ | A1 | | |
| | $\bar{x} = \frac{\int xy\,dx}{\int y\,dx} = \frac{\frac{32k}{5}}{4k}$ | M1 | | Forming equation to find $\bar{x}$ |
| | $= 1.6$ | A1F | 4 | ft 'their' part (a); must not contain $k$ |
| (c)(i) | $\frac{1}{2}\int y^2\,dx = \frac{1}{2}\int_0^2 k^2x^6\,dx = \left[\frac{k^2x^7}{14}\right]_0^2$ | M1 | | Attempt to use $\frac{1}{2}\int y^2\,dx$ |
| | $= \frac{64}{7}k$ | A1 | | |
| | $\bar{y} = \frac{\frac{1}{2}\int y^2\,dx}{\int y\,dx} = \frac{\frac{64k^2}{7}}{4k} = \frac{16k}{7}$ | M1 | | Finding $\bar{y}$ in terms of $k$ |
| | $\therefore \frac{16k}{7} = 8$ | | | |
| | $\therefore k = 3.5$ | A1F | 4 | ft 'their' part (a) |
| (ii) |  | | | |
| | $\tan\theta = \frac{\bar{y}}{\bar{x}}$ | M1 | | Use of $\tan\theta$ |
| | $= \frac{8}{1.6} = 5$ | A1F | | ft 'their' part (b); $\frac{\bar{y}}{\bar{x}}$ structure |
| | $\theta = 78.7°$ | A1F | 3 | |
| | **Total** | | **13** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | Moments about $C$: | B1 | | $P\cos\theta$ seen |
| | $mga = P\cos\theta a$ | M1 | | Forming moments equation – 2 terms |
| | $P = \dfrac{mg}{\cos\theta}$ | A1 | 3 | |
| **(b)** | Resolve ↔ $F = P\cos\theta$ 1 | M1 | | Resolve in two directions |
| | Resolve ↕ $mg = R + P\sin\theta$ 2 | A1 | | Both equations correct |
| | Friction law (sliding) $F = \mu R$ 3 | | | |
| | Substituting 1 and 2 in 3: | | | |
| | $P\cos\theta = \mu(mg - P\sin\theta)$ | m1 | | Substituting in $F = \mu R$ – dep on first M1 |
| | $P\cos\theta + P\mu\sin\theta = \mu mg$ | | | |
| | $P = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$ | A1 | 4 | AG |
| **(c)** | Slides first $\Rightarrow$ | | | |
| | $\dfrac{\mu mg}{\cos\theta + \mu\sin\theta} < \dfrac{mg}{\cos\theta}$ | M1 | | Set up inequality – expression in (b) < expression in (a)(ii) |
| | $\mu\cos\theta < \cos\theta + \mu\sin\theta$ | A1F | | Correct simplification – remove fractions ft parts (a) and (b) |
| | $\mu(\cos\theta - \sin\theta) < \cos\theta$ | | | |
| | $\mu < \dfrac{\cos\theta}{\cos\theta - \sin\theta}$ | A1 | 3 | CAO ; Alternative: $\mu < \dfrac{1}{1 - \tan\theta}$ |
| **(d)** | Inequality independent of mass, so no change | E2,1F | 2 | **No change** (E1) and **reason** (E1) ft error in (c); must give consistent reason If no reason, E0 |
| | **Total** | | **12** | |

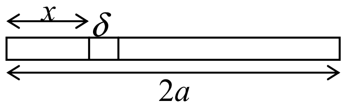
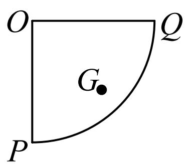
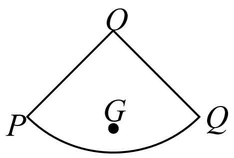
**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** | $\text{MI}_{\text{disc}} = \frac{1}{2}Mr^2 = 6mr^2$ | B1 | 1 | |
| **(b)(i)** |  | | | |
| | Using $C = I\ddot{\theta}$: | | | |
| | $T_1r - T_2r$ or $(T_1 - T_2)r$ | M1 | | Moments of both tensions seen |
| | $T_1r - T_2r = 6mr^2\ddot{\theta}$ | m1 | | Equation formed using $C = I\ddot{\theta}$ |
| | $T_1 - T_2 = 6mr\ddot{\theta}$ 1 | A1 | 3 | AG |
| **(ii)** | Equation of motion of *R*: | | | |
| | $T_2 = mr\ddot{\theta}$ 2 | M1<br>A1 | | Evidence of $r\ddot{\theta}$ anywhere<br>Correct equation |
| | Equation of motion of *P*: | | | |
| | $3mg - T_1 = 3mr\ddot{\theta}$ 3 | M1<br>A1 | | Attempt at F = *ma* – three terms<br>Correct equation |
| | Substituting 2 and 3 in 1:<br>$(3mg - 3mr\ddot{\theta}) - mr\ddot{\theta} = 6mr\ddot{\theta}$ | m1 | | Dep on previous M1 – solving three equations |
| | $3mg = 10mr\ddot{\theta}$ | | | |
| | $\ddot{\theta} = \frac{3g}{10r}$ | A1 | | AG |
| | **Alternative to (b)(ii)** | | | |
| | $\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}mv^2 + \frac{3}{2}mv^2 = 3mgh$ | (M1) | | Attempt at Conservation of Energy – three 'types' of term |
| | | (A1) | | Fully current equation |
| | $\frac{1}{2}(6mr^2)\dot{\theta}^2 + \frac{1}{2}m(r\dot{\theta})^2 + \frac{3}{2}m(r\dot{\theta})^2 = 3mg(r\theta)$ | (M1) | | $v = r\ddot{\theta}$ and $h = r\ddot{\theta}$ used |
| | $5r\dot{\theta}^2 = 3g\theta$ | (A1) | | Simplified to two terms $a\dot{\theta}^2 = b\theta$ |
| | Differentiate with respect to *t* | | | |
| | $10r\dot{\theta}\ddot{\theta} = 3g\dot{\theta}$ | (m1) | | Attempt to differentiate, dependent on first M1 |
| | $\ddot{\theta} = \frac{3g}{10r}$ | (A1) | 6 | |
| **(iii)** | $T_2 = mr\ddot{\theta} = \frac{3mg}{10r}$ | B1F | | ft 'their' equation for $T_2$ |
| | $T_1 = 3mg - 3mr\ddot{\theta} = 3mg - \frac{9mg}{10}$ | M1 | | Substituting in their equation for $T_1$ |
| | $= \frac{21mg}{10}$ | A1F | 3 | ft 'their' equation for $T_1$ |
| | **Total** | | **13** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6(a) | $\mathbf{r}_1 \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & 1 & 2 \\ \mathbf{j} & 0 & 0 \\ \mathbf{k} & 3 & a \end{vmatrix} = \begin{pmatrix} 0 \\ 6-a \\ 0 \end{pmatrix}$ | M1<br>A1 | | Attempt at $\mathbf{r}_1 \times \mathbf{F}_1$ – one comp correct<br>Fully correct |
| | $\mathbf{r}_2 \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & -1 & -2 \\ \mathbf{j} & 2 & 1 \\ \mathbf{k} & 0 & 3 \end{vmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$ | M1<br>A1 | | Attempt at $\mathbf{r}_2 \times \mathbf{F}_2$ – one comp correct |
| | $\begin{pmatrix} 0 \\ 6-a \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9-a \\ 3 \end{pmatrix}$ | A1 | 5 | Totalling $\mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$ ; AG<br><br>Note $\mathbf{F} \times \mathbf{r}$ scores M1A0M1A1A0 |
| (b)(i) | Magnitude of couple = 7 $\Rightarrow$ | | | |
| | $6^2 + (9-a)^2 + 3^2 = 7^2$ | M1 | | Forming equation – using part (a) |
| | $(9-a)^2 = 4$ | M1 | | Solving – to obtain two values |
| | $\therefore 9-a=2$ or $9-a=-2$<br>$a = 7$ or $a = 11$ | A1,A1 | 4 | AG (7) |
| | Special case (max 2):<br>If $a=7$, then resultant moment $= \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$ | | | |
| | Magnitude $= \sqrt{6^2 + 2^2 + 3^2} = 7$ | (M1)<br>(A1) | | |
| (ii) | $a = 7 \Rightarrow \mathbf{F}_1 + \mathbf{F}_2 = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ | M1 | | Attempt at $\mathbf{F}_1 + \mathbf{F}_2$ |
| | $= \begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}$ | A1 | 2 | $\mathbf{F}_1 + \mathbf{F}_2$ correct |
| | **Total** | | **11** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 7(a) |   | | | |
| | $\rho = \frac{m}{2a}$ | B1 | | $\rho$ seen anywhere |
| | MI for element $= (\rho\,\delta x)x^2$ | M1 | | Attempt at $mx^2$ |
| | MI rod $= \int_0^{2a} x^2 \rho\,\delta x = \int_0^{2a} \frac{mx^2}{2a}\,dx$ | | | |
| | $= \left[\frac{mx^3}{6a}\right]_0^{2a}$ | A1 | | Correct integration |
| | $= \frac{4}{3}ma^2$ | A1 | 4 | Correct use of units; AG |
| (b)(i) | $I_{OP} = I_{OQ} = \frac{4}{3}ma^2$ | | | |
| | $I_{\text{seat}} = 4m(2a)^2 = 16ma^2$ | M1A1 | | MI for seat – M1 for $mx^2$ form |
| | $\text{MI}_{\text{model}} = \frac{4}{3}ma^2 + \frac{4}{3}ma^2 + 16ma^2$ | M1 | | Sum of three MIs |
| | $= \frac{56ma^2}{3}$ | A1 | 4 | AG |
| (ii) |     | | | |
| | | | | Note $a = 1.5$ can be substituted anywhere |
| | KE gained $= \frac{1}{2}I\omega^2$ | M1 | | Use of $\frac{1}{2}I\omega^2$ |
| | $= \frac{28}{3}ma^2\omega^2$ | A1 | | |
| | PE lost $= 6mg(1.44a) - 6mg(1.44a\sin 45°)$ | M1 | | $mgh$ used |
| | $= 2.53mga$ | A1 | | Correct difference |
| | Conservation of energy $\Rightarrow$ | | | |
| | $\frac{28}{3}ma^2\omega^2 = 2.53mga$ | M1 | | Forming equation for C of E |
| | $a = 1.5 \Rightarrow \omega_{\max} = 1.33$ rad s$^{-1}$ | A1 | 6 | |
| | **Total** | | **14** | |
| | **TOTAL** | | **75** | |